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Transmittances and frequency characteristics of wave and diffusion heat transfer in the flat slab

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Abstract—This paper on conduction heat transfer in a slab presents a frequency domain comparison between the usual diffusion model and a wave model which includes the effect of non-instantaneous heat propagation. This is accomplished using a transfer function approach. Additionally, transfer functions relating the temperature distribution to heat flux are presented for various cases. The limit frequency, f_{li}, is developed as that frequency below which the difference between the diffusion model and the wave model is negligible. Above this frequency, the difference between the two models increases rapidly. The work of this paper is an essential generalization of various specific cases found in the previous literature. It also refers to previous papers written by the authors [*Int. J. Heat Mass Transfer* **36**, 1709–1713 (1993)].

1. INTRODUCTION

FOURIER's law models the dependence between the intensity of heat flux and the spatio-temporal distribution of temperature

$$\mathbf{q} = -k \operatorname{grad} T. \tag{1}$$

Combining equation (1) with the principle of energy conservation, we obtain the familiar parabolic heat equation

$$\alpha \nabla^2 T = \frac{\partial T}{\partial t}, \tag{2}$$

where $\alpha = k/\rho c$.

The physical interpretation of the solutions of equation (2) indicates an infinite heat propagation velocity, i.e. after excitation there is an immediate thermal reaction at any arbitrary point of the system. This often leads to a significant inaccuracy in the analysis of a number of thermal phenomena. A classical example here can be an intensive heating of solids by high energy thermal impulses of very short time duration. Energy fluxes of the type described above also appear during commutation of semiconductor elements [1] (e.g. thyristor) and are used in the annealing process by means of laser beams [2]. Other examples may include phenomena of high heat transfer in rarified media or in liquid and solid helium, etc.

Hence, under some circumstances, there is a need to remove the inaccuracy of the solution of equation (2). For this purpose we use a damped wave heat model. Fourier's law undergoes modification [3]

$$\mathbf{q} = -k \operatorname{grad} T - \tau \frac{\partial \mathbf{q}}{\partial t}. \tag{3}$$

By combining (3) with the principle of energy conservation we obtain a hyperbolic telegraph equation [4]

$$\alpha \nabla^2 T = \frac{\partial T}{\partial t} + \tau \frac{\partial^2 T}{\partial t^2}. \tag{4}$$

The solution of the above equation can be interpreted as a superposition of damped heat waves propagating at a finite velocity $a = \sqrt{(\alpha/\tau)}$.

Examination of the differences between the diffusion model equations (1), (2) and wave model equations (3), (4) is a very important task in basic research work.

Comparative analyses found in other papers refer mainly to semi-infinite media [5–8].

Due to the difficulty of taking reflected waves into consideration, the slabs are investigated less frequently [9, 10]. The slab configuration is understood here as a large, thin plate (Fig. 1).

It should be noted that in the literature, the analysis of the dynamics of heat transfer in the system described (semi-infinite and slab) has been limited to the time domain only. It is possible to obtain a qualitatively different view of the phenomenon by analysis in the frequency domain. By using properly defined operational transmittances, the spectral characteristics of the system can be determined. These spectral characteristics play a significant role in the analysis of various systems and very often constitute the best possible physical description. A good example of this is ref. [11], which describes probes for measuring wall fluctuation.

The present paper is a continuation of earlier papers by the authors [6].

2. BASIC DEFINITIONS AND TERMS

The slab considered here is heated by two heat fluxes penetrating both the front ($x = 0$) and the back ($x = h$) surfaces (Fig. 1). Generally accepted definition of transmittance requires an introduction of

NOMENCLATURE

- $A(s)$ any optional function for general solution of transform equation
- $A_i(x, \omega)$ amplitude characteristics of the slab with respect to the front side ($i = 1$) or back side ($i = 2$)
- a velocity of heat propagation in the wave model
- $B(s)$ any optional function for general solution of transform equation
- b dimensionless time of relaxation
- c specific heat of the slab
- $D_i(n, \mathrm{j}\beta)$ dimensionless spectral transmittance of the diffusion model with respect to the front side ($i = 1$) and back side ($i = 2$)
- $F_i(n, \mathrm{j}\beta)$ dimensionless spectral transmittance of the wave model with respect to the front side ($i = 1$) and back side ($i = 2$)
- f frequency
- grad $(\cdots)$ scalar function gradient
- h thickness of the slab
- j imaginary unit, $\sqrt{-1}$
- $K_i(x, s)$ operational transmittance of the slab with respect to the front side ($i = 1$) and back side ($i = 2$)
- $K_{fi}(x, s)$ operational transmittance of the wave model of the slab with respect to the front side ($i = 1$) and back side ($i = 2$)
- $K_{di}(x, s)$ operational transmittance of the diffusion model of the slab with respect to the front side ($i = 1$) and back side ($i = 2$)
- k thermal conductivity of the slab
- $k_i(x, t)$ impulse response of the slab with respect to the front side ($i = 1$) and back side ($i = 2$)
- m factor of proportionality between the heat fluxes penetrating the front side and the back side of the slab (specific case)
- n relative position of the point in the slab
- Q_0 heat flux amplitude
- $\mathbf{q}$ vector of the heat flux intensity at point x and time t, $q(x, t)\mathbf{1}x$
- $q_i(t)$ heat flux intensity penetrating the front side ($i = 1$) and back side ($i = 2$)
- $\bar{q}_i(s)$ Laplace transform of the heat flux penetrating the front side ($i = 1$) and back side ($i = 2$)
- $\bar{q}_R(s)$ Laplace transform of the heat flux at period R
- R period of heat flux intensity ($q_i(t) = q_i(t+R)$)
- s complex pulsation, $\mathrm{j}\omega$
- T space–time distribution of the temperature in the slab, $T(x, t)$
- $\bar{T}(x, s)$ Laplace transform of the temperature in the slab with respect to time
- t time
- t_0 duration of rectangular impulse
- x geometrical coordinate of the point in the slab.

Greek symbols

- α thermal diffusivity
- β dimensionless pulsation
- γ operational coefficient of heat transfer
- η apparent variable in the definite integral
- ρ density of the slab
- τ time of relaxation
- $\phi_i(x, \omega)$ phase characteristics of the slab with respect to the front side ($i = 1$) and back side ($i = 2$)
- ω pulsation.

Other symbols

- $\nabla^2(\cdots)$ scalar Laplacian operator
- $\mathbf{1}x$ unit vector in the OX direction.

zero initial conditions. Additionally the linearity of the medium has been assumed. The conditions mentioned above make it possible to formulate (on the basis of the superposition principle with respect to the heat fluxes—Fig. 1) a general form of the Laplace transform of the temperature [12]

$$\bar{T}(x, s) = K_1(x, s)\bar{q}_1(s) + K_2(x, s)\bar{q}_2(s). \quad (5)$$

Figure 2 shows a block diagram which illustrates equation (5). From the above equation we derive also the transmittance formula with respect to the front side:

$$K_1(x, s) \stackrel{\mathrm{df}}{=} \left.\frac{\bar{T}(x, s)}{\bar{q}_1(s)}\right|_{\bar{q}_2(s)=0}, \quad (6a)$$

and the back side

$$K_2(x, s) \stackrel{\mathrm{df}}{=} \left.\frac{\bar{T}(x, s)}{\bar{q}_2(s)}\right|_{\bar{q}_1(s)=0}. \quad (6b)$$

Thus defining the transfer function with respect to a given surface we assume that the opposite side is covered with a perfect thermal insulation. In spite of such an idealization the superposition of (6a) and (6b) corresponds to a real situation (Figs. 1 and 2).

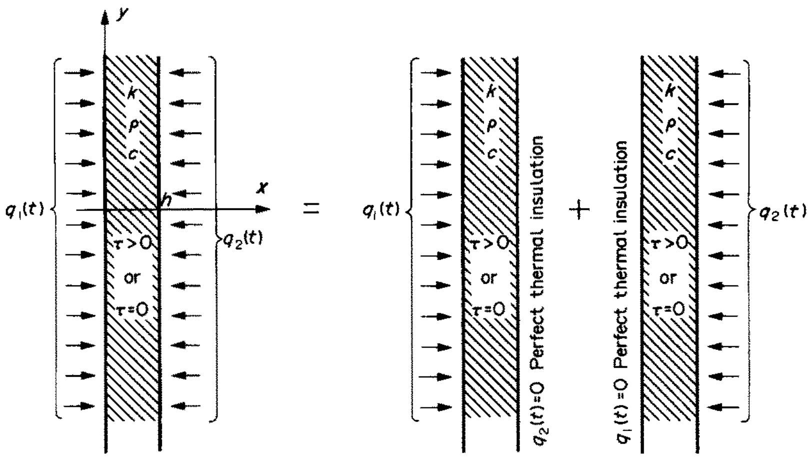

FIG. 1. Heat fluxes $q_1(t)$, $q_2(t)$ penetrating the slab of the following parameters: $k, \gamma, c, \tau > 0$ (wave model) and $\tau = 0$ (diffusion model). The superposition principle with respect to the fluxes.

Determining the transfer functions (6) is very useful for the following reasons:

(i) the dynamic properties of the system are determined independently of the form of the attacking heat fluxes $q_i(t)$,

(ii) the values of transfer functions on the imaginary axis define the amplitude characteristics

$$A_i(x,\omega) = |K_i(x, s = j\omega)|, \tag{7a}$$

and phase characteristics

$$\phi_i(x,\omega) = \arg K_i(x, s = j\omega), \quad \text{where } i = 1, 2. \tag{7b}$$

(iii) Borel's theorem makes it possible to determine the thermal response of the system to the excitation of any shape

$$T(x,t) = \sum_{i=1}^{2} \int_0^t k_i(x,\eta) q_i(t-\eta)\, d\eta. \tag{8}$$

The objective of the present study is to determine the transmittances (6) and the frequency characteristics (7) of the system shown in Fig. 1 for several heating scenarios.

3. TRANSFER FUNCTIONS OF THE SLAB

Due to the geometry of the system (Fig. 1), equation (4) takes the following form

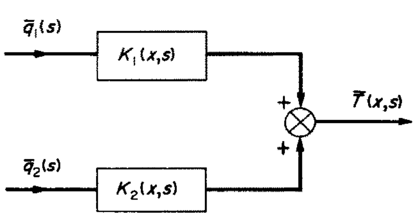

FIG. 2. Block diagram of the slab. $K_1(x, s)$—transmittance with respect to the front side. $K_2(x, s)$—transmittance with respect to the back side.

$$\alpha \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} + \tau \frac{\partial^2 T}{\partial t^2} \quad \text{for } 0 \leqslant x \leqslant h \quad \text{and } t \geqslant 0. \tag{9}$$

The boundary conditions are defined by the heating technique of the slab

$$\left.\begin{aligned} \mathbf{q}(0,t) &= q_1(t)\mathbf{1}x \\ \mathbf{q}(h,t) &= -q_2(t)\mathbf{1}x \end{aligned}\right\} \quad \text{for } t \geqslant 0. \tag{10}$$

Making use of equation (3), equation (10) can be represented in the scalar form

$$q_1(t) = -k \left.\frac{\partial T(x,t)}{\partial x}\right|_{x=0} - \tau \frac{dq_1(t)}{dt} \tag{11}$$

$$\text{for } t \geqslant 0.$$

$$-q_2(t) = -k \left.\frac{\partial T(x,t)}{\partial x}\right|_{x=h} + \tau \frac{dq_2(t)}{dt} \tag{12}$$

Considering the definition of the transfer function, zero initial conditions with respect to state variables (i.e. heat flux, temperature and temperature rate) have to be introduced

$$\mathbf{q}(x,0) = 0 \tag{13}$$

$$T(x,0) = 0 \qquad \text{for } 0 \leqslant x \leqslant h. \tag{14}$$

$$\left.\frac{\partial T(x,t)}{\partial t}\right|_{t=0} = 0 \tag{15}$$

A Laplace transformation with respect to time was performed on equation (9)

$$\alpha \frac{d^2 \bar{T}(x,s)}{dx^2} = s\bar{T}(x,s) - T(x,0) + s^2 \tau \bar{T}(x,s) - s\tau T(x,0) - \tau \left.\frac{\partial T(x,t)}{\partial t}\right|_{t=0}. \tag{16}$$

Applying the conditions of equations (13)–(15) gives

$$\frac{d^2 \bar{T}(x,s)}{dx^2} - \frac{s(1+s\tau)}{\alpha} \bar{T}(x,s) = 0. \tag{17}$$

Equation (17) is a second-order, linear, homogeneous differential equation in x, and the coefficients are independent of x. Hence, its solution is

$$\bar{T}(x,s) = A(s)\,\mathrm{ch}\,[\gamma(s)x] + B(s)\,\mathrm{sh}\,[\gamma(s)x], \quad (18)$$

$$\text{where } \gamma = \gamma(s) = \sqrt{\left(\frac{s(1+s\tau)}{\alpha}\right)}. \quad (19)$$

Selection of the positive real branch of equation (19) ensures a physical solution (i.e. positive temperatures) corresponding to $Re\, s \geqslant 0$ [13]

$$Re\sqrt{\left(\frac{s(1+s\tau)}{\alpha}\right)} \geqslant 0. \quad (20)$$

In order to specify the functions $A(s)$ and $B(s)$ we also perform a Laplace transformation of the boundary conditions (11), (12) with respect to t

$$\bar{q}_1(s) = -k\left.\frac{\mathrm{d}\bar{T}(x,s)}{\mathrm{d}x}\right|_{x=0} - s\tau\bar{q}_1(s) + \tau q_1(0), \quad (21)$$

$$-\bar{q}_2(s) = -k\left.\frac{\mathrm{d}\bar{T}(x,s)}{\mathrm{d}x}\right|_{x=h} + s\tau\bar{q}_2(s) - \tau q_2(0). \quad (22)$$

By virtue of equation (13) for $x = 0$ and $x = h$ and considering equation (10) the last terms of the right-hand sides of equations (21) and (22) are zero. Hence, the boundary conditions of equation (17) are

$$\left.\frac{\mathrm{d}\bar{T}(x,s)}{\mathrm{d}x}\right|_{x=0} = -\frac{1}{k}(1+s\tau)\bar{q}_1(s), \quad (23)$$

$$\left.\frac{\mathrm{d}\bar{T}(x,s)}{\mathrm{d}x}\right|_{x=h} = \frac{1}{k}(1+s\tau)\bar{q}_2(s). \quad (24)$$

By equating equations (23), (24) with the temperature gradient at either wall from equation (18), the functions $A(s)$ and $B(s)$ are determined

$$A(s) = \frac{(1+s\tau)}{k\gamma}\left[\frac{\mathrm{ch}\,(\gamma h)}{\mathrm{sh}\,(\gamma h)}\bar{q}_1(s) + \frac{1}{\mathrm{sh}\,(\gamma h)}\bar{q}_2(s)\right], \quad (25)$$

$$B(s) = -\frac{(1+s\tau)}{k\gamma}\bar{q}_1(s), \quad (26)$$

where the abbreviated notation $\gamma = \gamma(s)$ is used. The expression for $A(s)$ and $B(s)$ from equations (25) and (26) are now inserted back into equation (18), yielding an expression for the Laplace transformed temperature in terms of material parameters and the Laplace transformed heat fluxes, $\bar{q}_1(s)$, $\bar{q}_2(s)$. Then, from the transfer function definitions of the transmittances, equations (6a) and (6b), we arrive at

$$K_{\mathrm{f}1}(x,s) = \frac{(1+s\tau)}{k\gamma(s)}\,\frac{\mathrm{ch}\,[\gamma(s)(h-x)]}{\mathrm{sh}\,[\gamma(s)h]}, \quad (27)$$

$$K_{\mathrm{f}2}(x,s) = \frac{(1+s\tau)}{k\gamma(s)}\,\frac{\mathrm{ch}\,[\gamma(s)x]}{\mathrm{sh}\,[\gamma(s)h]}, \quad (28)$$

where $\gamma(s)$ is defined by equation (19) with the constraint of equation (20). Relation (27) represents the transfer function with respect to the front side of the slab, whereas equation (28) represents the function with respect to the back side of the slab.

4. SOME SELECTED CASES OF TRANSMITTANCE OF THE SLAB

In a number of specific cases it is possible to introduce (based on relations (27), (28) and the block diagram from Fig. 2) one element transfer function with respect to the front side of the slab

$$K_{\mathrm{f}}(x,s) = K_{\mathrm{f}1}(x,s) = \frac{\bar{T}(x,s)}{\bar{q}_1(s)}, \quad (29)$$

which greatly simplifies definitions (6).

In the simplification process we make use of various hyperbolic trigonometric identities given in ref. [14].

Equal heating of the slab from both sides, i.e. $\bar{q}_2(s) = \bar{q}_1(s)$

$$K_{\mathrm{f}}(x,s) = \frac{(1+s\tau)}{k\gamma(s)}\,\frac{\mathrm{ch}\,[\gamma(s)(x-0.5h)]}{\mathrm{sh}\,[0.5\gamma(s)h]}. \quad (30)$$

Zero net heat transfer to the slab, i.e. $\bar{q}_2(s) = -\bar{q}_1(s)$

$$K_{\mathrm{f}}(x,s) = -\frac{(1+s\tau)}{k\gamma(s)}\,\frac{\mathrm{sh}\,[\gamma(s)(x-0.5h)]}{\mathrm{ch}\,[0.5\gamma(s)h]}. \quad (31)$$

Proportional heat fluxes penetrating the slab, i.e. $\bar{q}_2(s) = -m\bar{q}_1(s)$

$$K_{\mathrm{f}}(x,s) = \frac{(1+s\tau)}{k\gamma(s)}\,\frac{\mathrm{ch}\,[\gamma(s)(h-x)] - m\cdot\mathrm{ch}\,[\gamma(s)x]}{\mathrm{sh}\,[\gamma(s)h]}. \quad (32)$$

where m = constant.

Heating of the front side by the operational heat flux $\bar{q}_1(s)$ *with adiabatic back side, i.e.* $\bar{q}_2(s) = 0$

In such a case transmittance is expressed by equation (27). After expressing the hyperbolic functions in exponential form and multiplying both the numerator and denominator by exp $(-\gamma h)$ we obtain

$$K_{\mathrm{f}}(x,s) = \frac{(1+s\tau)}{k\gamma}\,\frac{\mathrm{e}^{-\gamma x} + \mathrm{e}^{-\gamma(2h-x)}}{1-\mathrm{e}^{-2\gamma h}}, \quad (33)$$

where $\gamma = \gamma(s)$. It can be easily noted that equation (33) can be used as a basis for obtaining all the results from ref. [9]. For this purpose it is enough to introduce in equation (29)

$$\bar{q}_1(s) = Q_0 \quad \text{or} \quad \bar{q}_1(s) = \frac{Q_0}{s}(1-\mathrm{e}^{-st_0})$$

for Dirac and rectangular impulses respectively.

Semi-infinite medium ($h \to \infty$)

Using $\gamma(s)$ as given by equation (19) with the constraint of equation (20), ensures mathematical correctness of the conversion from the slab to semi-infinite medium ($h \to \infty$, cf. Fig. 1). In such a case we should take the following into consideration:

$$\lim_{h\to\infty} e^{-\gamma(2h-x)} = \lim_{h\to\infty} e^{-2\gamma h} = 0.$$

Having made the necessary transformation in equation (33) we have

$$K_f(x,s) = \frac{\sqrt{(\alpha\tau)}}{k}\frac{\sqrt{\left(s+\frac{1}{\tau}\right)}}{\sqrt{s}} \times \exp\left(-\frac{\sqrt{\left(s\left(s+\frac{1}{\tau}\right)\right)}}{a}x\right). \quad (34)$$

Equation (28) tends to zero as $h \to \infty$ upon employing the constraint of equation (20). Physically, this is because $q_2(t) = 0$. Equation (34) was also obtained in an earlier work done by the authors [6]. The formula (34) may also be used as a basis for the work [5] dealing with linear aspects of heat transfer. To obtain the results [5] it is necessary to substitute into equation (29):

$$\bar{q}_1(s) = \frac{\bar{q}_R(s)}{1-e^{-sR}}.$$

The above remarks also refer to article [8] if we express $\bar{q}_1(s)$ as a sinusoidal heat flux

$$\bar{q}_1(s) = Q_0\frac{\omega}{s^2+\omega^2}.$$

Transmittances of diffusion heat flow ($\tau \to 0$)

When the thermal relaxation time tends to zero the diffusion of heat becomes a limiting case of wave heat propagation. Then the propagation velocity $\sqrt{(\alpha/\tau)}$ becomes infinite. The relation (3) then agrees with Fourier's law (1). In equation (4) the term responsible for the wave character of the phenomenon disappears. In the above case, on the basis of equations (27) and (28), we can determine the transfer function of the slab described by the diffusion equation

$$K_{di}(x,s) = \lim_{\tau\to 0} K_{fi}(x,s), \quad (35)$$

where $i = 1, 2$.

From equation (35) we obtain the desired transfer function of heat diffusion with respect to the front side of the slab

$$K_{d1}(x,s) = \frac{1}{k}\frac{\operatorname{ch}\left[\sqrt{\left(\frac{s}{\alpha}\right)}(h-x)\right]}{\sqrt{\left(\frac{s}{\alpha}\right)}\operatorname{sh}\left(\sqrt{\left(\frac{s}{\alpha}\right)}h\right)}, \quad (36)$$

and with respect to the back side

$$K_{d2}(x,s) = \frac{1}{k}\frac{\operatorname{ch}\left(\sqrt{\left(\frac{s}{\alpha}\right)}x\right)}{\sqrt{\left(\frac{s}{\alpha}\right)}\operatorname{sh}\left(\sqrt{\left(\frac{s}{\alpha}\right)}h\right)}. \quad (37)$$

Similarly, the results of equation (34) for a semi-infinite medium become, in the limit as $\tau \to 0$,

$$K_d(x,s) = \frac{\sqrt{(\alpha)}}{k}\frac{1}{\sqrt{(s)}}\exp\left(-x\sqrt{\left(\frac{s}{\alpha}\right)}\right), \quad (38)$$

which remains in good agreement with the results of ref. [6].

5. FREQUENCY CHARACTERISTICS OF THE SLAB

The amplitude and phase of the transmittances are found by applying equation (7) to the transmittances derived for various cases of Sections 3 and 4. The transmittances are converted to the frequency domain by the substitution $s = j\omega$. For convenience, the transmittances will be recast in nondimensional form. This is accomplished by multiplying the transmittances by k/h and performing the substitutions:

$$n = x/h,$$
$$b = \tau\alpha/h^2$$
$$\beta = \omega h^2/\alpha,$$

where $h^2/\alpha = \text{const}$. Hence, equations (27) and (28) become

$$F_1(n, j\beta) = \frac{k}{h}K_{f1}\left(\frac{x}{h}, j\omega\frac{h^2}{\alpha}\right) = \frac{\sqrt{(1+jb\beta)}\operatorname{ch}[(1-n)\sqrt{(j\beta)}\sqrt{(1+jb\beta)}]}{\sqrt{(j\beta)}\operatorname{sh}[\sqrt{(j\beta)}\sqrt{(1+jb\beta)}]}, \quad (39)$$

and with respect to the back side

$$F_2(n, j\beta) = \frac{k}{h}K_{f2}\left(\frac{x}{h}, j\omega\frac{h^2}{\alpha}\right) = \frac{\sqrt{(1+jb\beta)}\operatorname{ch}[n\sqrt{(j\beta)}\sqrt{(1+jb\beta)}]}{\sqrt{(j\beta)}\operatorname{sh}[\sqrt{(j\beta)}\sqrt{(1+jb\beta)}]}. \quad (40)$$

In the case of the diffusion model the above relations are simplified ($b = 0$) and assume the following form

$$D_1(n, j\beta) = \frac{k}{h}K_{d1}\left(\frac{x}{h}, j\omega\frac{h^2}{\alpha}\right) = \frac{1}{\sqrt{(j\beta)}}\frac{\operatorname{ch}[(1-n)\sqrt{(j\beta)}]}{\operatorname{sh}(\sqrt{(j\beta)})} \quad (41)$$

with respect to the front side, and

$$D_2(n, j\beta) = \frac{k}{h}K_{d2}\left(\frac{x}{h}, j\omega\frac{h^2}{\alpha}\right) = \frac{1}{\sqrt{(j\beta)}}\frac{\operatorname{ch}(n\sqrt{(j\beta)})}{\operatorname{sh}(\sqrt{(j\beta)})} \quad (42)$$

with respect to the back side.

In order to present the slab characteristics graphically, relations (39)–(42) were tabulated for the pos-

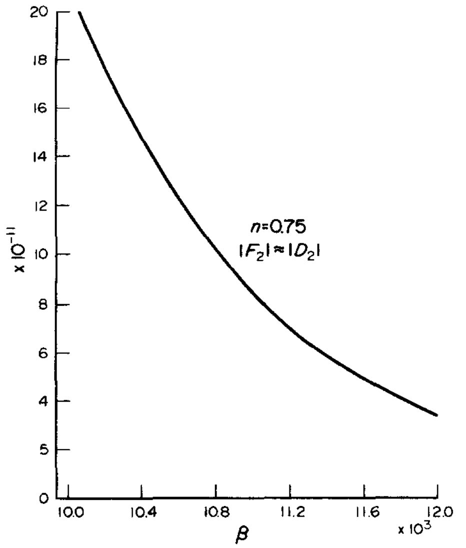

Fig. 3. Combined amplitude characteristics of wave $|F_2|$ and diffusion $|D_2|$ heat transfer with respect to the back side of the slab for the low values of dimensionless pulsation β in asymmetrical position ($n = 0.75$).

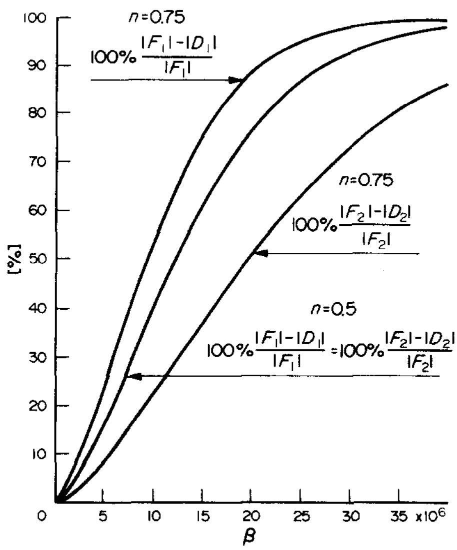

Fig. 5. Relative difference between amplitude characteristics of wave ($|F_1|$, $|F_2|$) and diffusion ($|D_1|$, $|D_2|$) heat transfer as dimensionless pulsation function in symmetrical position ($n = 0.5$) and asymmetrical position ($n = 0.75$) with respect to the front side ($|F_1|$, $|D_1|$) and back side ($|F_2|$, $|D_2|$).

itions in the centre of the slab ($n = 0.5$) and at $n = 0.75$.

A typical solid parameters were assumed:

$$k = 145\ \mathrm{W\,m^{-1}\,K^{-1}}, \quad c = 700\ \mathrm{J\,kg^{-1}\,K^{-1}},$$

$$\rho = 2330\ \mathrm{kgm^{-3}}, \quad \tau = 10^{-10}\ \mathrm{s}, \quad h = 0.01\ \mathrm{m}.$$

Secondary parameters for the above values are:

$$\alpha = 8.89 \times 10^{-5}\ \mathrm{m^2\,s^{-1}}, \quad a = 942.87\ \mathrm{m\,s^{-1}},$$

$$b = 8.89 \times 10^{-11}.$$

The results obtained here were presented graphically in Figs. 3–5.

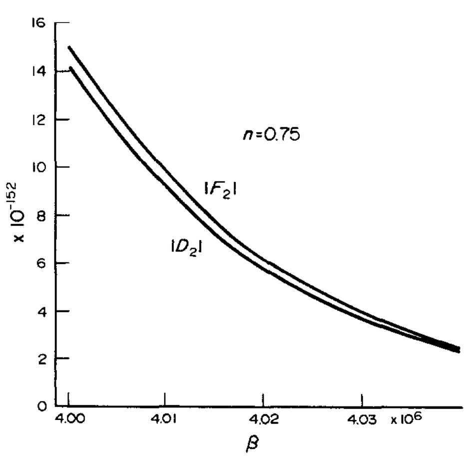

Fig. 4. Amplitude characteristics of wave $|F_2|$ and diffusion $|D_2|$ heat transfer with respect to the back side for higher values of dimensionless pulsation β in asymmetrical position ($n = 0.75$).

Figure 3 shows the characteristics within the range of low and medium frequencies. As can be seen the wave model is practically identical with the diffusion model. It results from small values of the relaxation time in solids (10^{-12}–10^{-4} s) [15].

The effect of thermal relaxation is manifested at higher frequencies causing an increase of the temperature field in the slab. Hence for the fast changing signals in time the values of the amplitude characteristics of the wave model are greater than in the diffusion model (Fig. 4).

Relative differences between the hyperbolic and parabolic characteristics are presented in Fig. 5. These differences are the function of nondimensional pulsation β. Assuming a 3% difference between the characteristics values we obtained (for $n \approx 1$) a nondimensional limit pulsation $\beta_{li} = 1.2 \times 10^6$ ($f_{li} = 170$ kHz). Above this limit the difference begins to increase rapidly and at $\beta = 40 \times 10^6$ ($f = 5.7$ MHz), reaches almost 100% (Fig. 5).

6. CONCLUSIONS

The present paper is a continuation of the article [6] and deals with the problems in the frequency domain which so far have been described in the time domain only (refs. [5, 7, 9, 10]). The transmittances (27), (28), (36), (37) determined in the paper make it possible to describe the Laplace transform (and next the original) of the temperature at each point of the analysed slab for any arbitrary excitation heat fluxes. The above transfer functions are irrational and hyperbolic relations of complex pulsation $s = j\omega$. Such a non-algebraic form is typical of distributed parameter systems.

Comparing (27) with (36) and (28) with (37) we notice that the respective expressions in front of the fraction line and the arguments of the hyperbolic function differ in the factor

$$\sqrt{(1+s\tau)}\big|_{s=j\omega} = \sqrt{(1+j\omega\tau)}.$$

The above is the cause of appearing in the hyperbolic model of temperature increases, delay of wave motion and wave reflections. They are definitely different effects than the ones occurring in the parabolic model.

An increase of relaxation time causes a drop in the velocity of heat propagation and therefore a lower limit pulsation β_{li} (see Section 5). Hence, for such a case, we have an increase in the importance of accounting for the wave properties of the system.

The diagram of frequency characteristics (Figs. 3–5) was described in Section 5. Formulas (39)–(42) point to interdependence of the characteristics on relative position n, dimensionless time of relaxation b, and on dimensionless pulsation β. Making use of these parameters it is possible to determine harmonically variable steady field for β = const. It is also possible to determine transient thermal fields, because in papers [16], [17] calculation methods of transient states on the basis of frequency characteristics had been described. Hence spectral transmittances (39)–(42) give a complete picture of the slab dynamics.

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